A Modified Dual-Wavelength Technique for Ku- and Ka-band Radar Rain Retrieval

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1 **Abstract**
2
3   To overcome a deficiency in the standard Ku- and Ka-band dual-wavelength radar

4 technique, a modified version of the method is introduced. The deficiency arises from

5 ambiguities in the estimate of the mass-weighted diameter (D_m) of the raindrop size distribution

6 (DSD) derived from the differential frequency ratio (DFR), defined as the difference between the

7 radar reflectivity factors in dB at Ku- and Ka-band, Z_{Ku}-Z_{Ka}. In particular, for DFR values less

8 than zero, there are two possible solutions of D_m, leading to ambiguities in the retrieved DSD

9 parameters. It is shown that the double solutions to D_m are effectively eliminated if the DFR is

10 modified from Z_{Ku}-Z_{Ka} to Z_{Ku}-γZ_{Ka} (in dB) where γ is a constant with a value less than 0.8. An

11 optimal radar algorithm that uses the modified DFR for the retrieval of rain and D_m profiles is

12 described. The validity and accuracy of the algorithm are tested by applying it to radar profiles

13 that are generated from measured DSD data. Comparisons of the rain rates and D_m estimated

14 from the modified DFR algorithm to the same hydrometeor quantities computed directly from

15 the DSD spectra (or the truth) indicate that the modified DFR-based profiling retrievals perform

16 fairly well, and are superior in accuracy and robustness to retrievals using the standard DFR.

17

1. Introduction

1   There is an increasing trend of using multiple wavelength radars in an effort to accurately

2   detect and retrieve cloud and precipitation microphysical properties. The dual-wavelength radar

3   is, however, the most popular among these applications. It makes use of two wavelengths, one of

4   which is such that Rayleigh scattering dominates for most precipitation-sized particles and the

5   other short enough so that non-Rayleigh (or Mie) scattering occurs in the presence of large

6   hydrometeors. For precipitation measurements, the combination of Ku- (or X-) and Ka-band is

7   found in many air/space-borne radar applications. This choice is dictated not only by the desire

8   for microphysical information but by practical considerations such as frequency allocation (for

9   space-based radars), the advantages of matched radar beams, and the desire for good spatial

10   resolution within the constraints of antenna size. Attenuation caused by rain and mixed-phase

11   hydrometeors, among other factors, is also important in choosing frequencies not only because it

12   adds difficulty in estimating precipitation but because it often prevents the radar signal from

13   penetrating the entire storm.

14   The Dual-wavelength Precipitation Radar (DPR) operating at Ku- and Ka-band, aboard the

15   Global Precipitation Measurement (GPM) core satellite, is the first dual-wavelength radar in

16   space (Hou et al. 2008). An important goal of the DPR is to derive rain rate and raindrop size

17   distribution (DSD), the latter of which is often modeled by an analytical function (Ulbrich, 1983;

18   Gorgucci et al. 2000 and 2002; Bringi et al. 2002; Feingold and Levin 1986). Several dual-

19   wavelength radar retrieval techniques have been developed which are based on the standard

20   dual-wavelength technique that uses the differential frequency ratio (DFR), i.e., the difference of

21   radar reflectivities between two wavelengths, to first infer the DSD parameters, correct

22   attenuation and then derive the rain rate profile gate by gate either stepping forward or backward

1   along the radar beam (Meneghini et al. 1992 and 1997; Mardiana et al. 2004; Liao and

2   Meneghini 2005; Rose and Chandrasekar 2005; Seto et al. 2013; Seto and Iguchi 2015; Liao et

3   al. 2016). To improve the robustness of the dual-wavelength retrieval, several optimal estimation

4   methods have been recently proposed, including the GPM DPR (version-5) algorithm, in which

5   one or more adjustment factors are used to modify nominal relationships between the parameters

6   of the radar and the precipitation (such as k-Z and R-D_m relations, where k, Z, R and D_m are

7   specific attenuation, radar reflectivity, rain rate and mass-weighted diameter) to account for

8   variations in the DSD in space and time (Grecu et al. 2011; Seto and Iguchi 2015). Adjustment

9   factors at each gate/profile are determined by optimizing predefined cost functions that constrain

10   the values of the reflectivities and path integral attenuations (PIA).

11   In the standard DFR-based dual-wavelength technique the two unknown DSD parameters,

12   related to the characteristic size and number density of raindrops, are estimated at each range

13   gate. However, this is successful only if there exist unique solutions of the DSD parameters to

14   the dual-wavelength radar equations. For the gamma DSD model, ambiguities in the estimation

15   of D_m often occur, particularly for light-to-moderate rain rates. As there is no additional

16   information aiding in selection of the solution, the standard dual-wavelength technique becomes

17   problematic for rain retrievals. Retrieval errors caused by improper selection of the solution

18   worsen as retrieval proceeds deep into the storm as a result of error propagation. It has been

19   revealed in our study from a large collection of the measured DSD data from various NASA-

20   sponsored field campaigns that negative DFR values occur frequently. Since these negative DFR

21   values lead to ambiguities in the estimate of D_m and therefore N_w and further that these errors are

22   propagated in range because of attenuation correction, the overall performance of the dual-

23   frequency retrieval will be adversely affected.

1   To circumvent the double-value problem of D_m, a modified DFR, denoted by DFR*, is

2   introduced in this study. Instead of weighting equally the Ku- and Ka-band reflectivities, the

3   DFR* weights the Ka-band reflectivity less than the Ku-band so that the possibility of double

4   solutions diminishes. Removal of the double values of D_m with use of DFR* aids in finding D_m

5   unambiguously from the D_m-DFR* relations; the trade-off, however, is that the DFR* is no

6   longer exclusively a function of D_m but a function of the number concentration parameter as

7   well.

8   In this study, our focus is on examining the potential of DFR* for the retrieval of the

9   vertical rain and DSD profiles by employing measured DSD data. For understanding the

10   uncertainties in rain rate estimation and also in gaining insight into ways to improve the

11   algorithms, a physical evaluation of the standard (DFR-based) and modified (DFR*-based) dual-

12   wavelength techniques is conducted by applying them to simulated vertical rain profiles. The

13   hydrometeor profiles for use in our evaluation are constructed by using the measured DSD

14   acquired from various storm systems.

15   In this paper we begin with briefly describing the principle and issues associated with the

16   standard dual-wavelength technique in Section 2, followed by the discussion of the modified

17   dual-wavelength technique in Section 3. Described in Section 4 are the methodology related to

18   the radar profiling algorithms and their procedures for the retrieval of rain and DSD profiles

19   using the DFR and DFR* while evaluation of the performance of the profiling algorithms is

20   made in Section 5. Summary and remarks are given in Section 6.

21   **2.**   **Standard Dual-Wavelength Radar Technique**

22

23   The measured radar reflectivity factor (z_m in mm^6/m^3) of the hydrometeors at range r and at

24   wavelength λ can be expressed as

1 $$z_m = z_e \exp(-0.2\ln(10) \int_0^r k\,dr),$$ (1)

2 where z_e is the effective radar reflectivity factor (mm^6/m^3) and k is specific attenuation (dB/km),

3 expressed as

4 $$z_e = \frac{\lambda^4}{\pi^5 |K_w|^2} \int_0^\infty N(D)\sigma_b(D,\lambda)\,dD$$ (2)

5 and

6 $$k = 4.343 \times 10^{-3} \int_0^\infty N(D)\sigma_e(D,\lambda)\,dD,$$ (3)

7 where $\sigma_b(D,\lambda)$ and $\sigma_e(D,\lambda)$ are radar backscattering and extinction cross sections (mm^2) of a

8 raindrop with volume equivalent diameter D (mm) at a wavelength of λ (mm), respectively.

9 N(D) (mm^{-1}m^{-3}) is the gamma DSD and expressed as (Ulbrich 1983; Bringi and Chandrasekar

10 2001)

11 $$N(D) = N_w f(\mu) \left(\frac{D}{D_m}\right)^\mu \exp(-\Lambda D),$$ (4)

12 where N_w is a scale factor, and

13 $$f(\mu) = \frac{6(4+\mu)^{\mu+1}}{4^4 \Gamma(\mu+4)},$$ (5)

14 and

15 $$\Lambda = (4+\mu)/D_m,$$ (6)

16 where Γ is the Gamma function, and μ is the shape factor of the gamma distribution. D_m is the

17 mass-weighted diameter of particle spectra, defined as follows

18 $$D_m = \frac{\int_{D_{min}}^{D_{max}} D^4 N(D)\,dD}{\int_{D_{min}}^{D_{max}} D^3 N(D)\,dD},$$ (7)

19 D_{min} and D_{max} are the minimum and maximum diameters of raindrops, respectively. The rain rate

20 is computed by

21 $$R = 6\pi \times 10^{-4} \int_{D_{min}}^{D_{max}} N(D) \cdot D^3 V(D)\,dD,$$ (8)

1   where V(D) is raindrop fall velocity and expressed as a function of D.

2   Because the radar reflectivity factor usually spans a large range of magnitudes, it is

3   conveniently expressed in decibels (dB) of reflectivity or dBZ as follows

4 $$Z = 10\log_{10} z. \tag{9}$$

5   Note that lower z and upper case Z denote linear and logarithmic reflectivities, respectively,

6   throughout this study. The radar differential frequency ratio (DFR) in dB is defined as

7 $$DFR = Z_{Ku} - Z_{Ka}, \tag{10}$$

8   where Z_{Ku} (dBZ) and Z_{Ka} (dBZ) are the radar reflectivity factors at wavelengths of Ku and Ka

9   bands. From (2), (4) and (10) it is not difficult to show that DFR is independent of N_w. Using

10   (4), Fig.1 shows D_m (left) and the z_{Ku}-normalized rain rate (right) as a function of DFR for

11   several μ values. Note that the one-to-one relationship between D_m and DFR when DFR is

12   greater than zero, implying that, if μ is known and the DFR is positive, D_m can be determined

13   uniquely. However, double solutions of D_m occur as the DFR becomes negative. This has also

14   been found by many earlier studies (Meneghini et al. 1992 and 1997; Mardiana et al. 2004; Liao

15   and Meneghini 2005; Rose and Chandrasekar 2005; Seto et al. 2013; Seto and Iguchi 2015).

16   Similar features are found in the normalized rain rate plots shown in the right panel of Fig.1,

17   where a unique solution to R/z_{Ku} is available only when the DFR is positive.

18   The axes of the plots in Fig.2 are the same as in Fig. 1 but in this case, the results,

19   displayed as 2-dimensional probability density functions (PDF), are derived from measured

20   DSDs acquired during the Iowa Flooding Studies (IFloodS), the Integrated Precipitation

21   Validation Experiment (IPHEx) and data from NASA Wallops Flight Facility in Wallops

22   Island, Virginia. For the sake of comparisons with model results, theoretical computations

23   from the gamma DSD model with a fixed μ of 3 are given by the white curves. It is worth

1   mentioning that use of the z_{Ku}-normalized rain rate, as in Figs.1 and 2, is a convenient and

2   effective way to directly infer rain rate from radar parameters. In particular, given a (positive)

3   DFR and an assumed μ, the R/z_{Ku} is obtained from the curves in the figure; multiplication of the

4   result by z_{Ku} then provides the rain rate estimate. N_w is obtained from either Ku- or Ka-band

5   reflectivities along with the derived D_m. For the scattering computations, the raindrops are

6   prescribed as horizontally oriented oblate spheroids with axis ratios that follow the shape-size

7   relations reported by Thurai et al. (2007). Symmetrical axes of the raindrops are aligned in the

8   vertical direction. The T-matrix method is used to compute the scattering properties of single

9   particles (Mishchenko and Travis 1998). It is important to point out that vast majority of the

10   DSD measurements (nearly 70%), as found in Fig.2, fall into the range where the DFR is

11   negative, which suggests a serious problem for operational retrieval algorithms if the DFR-D_m

12   relations are used. For the GPM DPR whose minimum detectable reflectivities are approximately

13   12 dB and 17 dB at Ku- and Ka-wavelength, respectively, the percentage of the DSD data with

14   the DFR less than 0 is around 37%.

15   **3.   Modified dual-wavelength Radar Technique**

16   As discussed above, the standard dual-wavelength algorithm is capable of retrieving two

17   parameters of the DSD and rain rate at each range gate if the fixed-μ gamma DSD model is

18   assumed and if the DFR is positive along the path. Apart from errors in the DSD model assumed,

19   the most serious problem arises from ambiguities in the DFR-D_m and DFR-R/Z_{Ku} relationships

20   when the DFR is negative. To lessen the impact of these ambiguities, a modified DFR, denoted

21   by DFR*, is introduced and expressed as

22   $$DFR^* = Z_{Ku} - \gamma Z_{Ka}, \tag{11}$$

1   where γ is a scale factor with a value ranging from 0 to 1. The definition of DFR* in (11) can be

2   viewed as a generalized form of the dual-wavelength ratio which reverts to the single-

3   wavelength Z_{Ku} when $\gamma=0$ and to the standard dual-wavelength method when $\gamma=1$. For γ greater

4   than 0 but less than 1, the Ka-band radar reflectivity is used but reduced from that in the DFR

5   relation.   Unlike the DFR, the DFR* is dependent on N_w.

6   Figure 3 illustrates the DFR*-D_m relation, derived from the DSD data, as γ takes on

7   different values. The thick black solid lines are the means of the data taken at each point along

8   the horizontal axis. Theoretical computations (blue curves) derived from the gamma DSD model

9   with a fixed μ of 3 are plotted for N_w varying from 10 to 10^5 $mm^{-1}m^{-3}$. For reference, the

10   contours of Z_{Ku} in dBZ (red curves) are also given in the plots. Shown in the top-left panel is the

11   case for the single Ku-band, i.e., $\gamma=0$, while the results from the standard dual-wavelength

12   technique ($\gamma=1$) are shown in the bottom-right panel. The cases where γ is equal to 0.5 and 0.7

13   are displayed in the top-right and bottom-left panels, respectively. Several interesting features are

14   revealed in Fig.3; in particular, D_m has one-to-one relation with DFR* for all the cases shown

15   except for the case where $\gamma=1$. As expected, the DFR*-D_m relations depend on N_w if γ is not 1,

16   and the degree of the dependence depends on γ. The strongest dependence of N_w on these

17   relations occurs at $\gamma=0$ (Ku-band only), as shown from both the DSD-derived data points and the

18   model computations. Variability in the DFR*-D_m with respect to N_w tends to decrease for D_m

19   greater than 1 mm as γ increases. The DFR* at $\gamma=1$, or the standard DFR, is independent of N_w.

20   It is worth reiterating that one of the crucial differences between the DFR* ($\gamma\neq1$) and the

21   standard DFR is that the former gives a unique solution to D_m for given N_w and μ while the latter

22   results in double values of D_m for large amount of the radar measurements. Similar behavior is

23   also found in the rain rate estimates, as depicted in Fig.4. Unique solutions to D_m and rain rate

1   from the DFR* are perhaps the most desirable features from the perspective of radar retrievals,

2   but it comes with the cost of adding the variable N_w to the equations and leading to an under-

3   constrained retrieval. As one of the goals of the spaceborne radar is to obtain hydrometeor

4   profiles along the radar beam, the rest of the paper is focused on how the modified dual-

5   wavelength technique described above is implemented in the profiling algorithms and how its

6   performance compares with algorithms that rely on the standard dual-wavelength technique.

7   **4.   Hydrometeor profile retrieval**

8   Although the modified dual-wavelength method or DFR* shows potential to improve

9   accuracy of radar estimates over the standard DFR in terms of the uncertainties associated with

10   double solutions, a physical evaluation of the hydrometeor profiling retrieval is required not

11   only for understanding the uncertainties but also in gaining insight into ways to improve

12   the retrievals. To achieve this, the measured DSD data acquired from a variety of storm

13   systems during the NASA field campaigns are used to construct realistic hydrometeor

14   profiles. Using measured DSD spectra both radar and hydrometeor's parameters, which

15   include true and measured radar reflectivities, attenuation, rain and DSD characteristic

16   parameters (D_m and N_w), can be computed. These DSD-calculated parameters then serve as

17   truth. An evaluation of algorithm performances is actually conducted by comparing the

18   radar estimates with the truth. Before showing the results on the DFR* profiling algorithm

19   performance, we will first describe the radar and hydrometeor profiles simulated by using

20   measured DSD, and then detail the procedures of the DFR* profile algorithms.

21   4.1   DSD profiles

22   For simplicity but without losing generality, two types of vertical DSD profiles are

23   created: One type is the vertically uniform DSD profile where a single DSD spectrum is used for

1   entire column, and second type is the non-uniform profile in which the DSD changes gate by

2   gate along the column. The DSD data used in this study are primarily from measurements made

3   by Parsivel disdrometers during the Iowa Flood Studies (IFloodS) field experiment from May 1

4   to June 15, 2013, the Integrated Precipitation Validation Experiment (IPHEx) from May 1 to

5   June 15, 2014, and the observations at the NASA Wallops Flight Facility in Wallops Island,

6   Virginia from 2013 to 2014. The Parsivel disdrometers are used to measure rain DSD, and fall-

7   velocities of particle sizes from 0.3 to 20 mm.

8   Shown in Fig.5 is an example of the simulated Ku-band radar reflectivities (top panels)

9   and rain rates (bottom panels) from a segment of 200 DSD vertically uniform (left column) and

10   non-uniform (right column) profiles. The range resolution is 0.125 km, which is comparable to

11   the case for the GPM DPR, with the rain height set at 5 km. To construct realistic non-uniform

12   hydrometeor profiles, the time-series DSD data are converted to vertical profiles by assigning 1-

13   minute-integration-time DSD data to the gates from the top to surface consecutively. The mean

14   values of the correlations of rain, D_m and $\log_{10}N_w$ over entire simulated non-uniform profiles

15   between the range gates considered to the surface gate, are plotted in Fig.6 for the cases in which

16   rain rate is less than 5 mm/h (left), greater than 5 mm/h (middle) and greater than zero (right),

17   which approximately represent the stratiform, convective and stratiform-convective-combined

18   rain systems. These correlations are found to be comparable to the results reported by Adirosi et

19   al. (2016) based on a combined study of Micro Rain Radar (MRR) and DSD measurements.

20   4.2   Modified DFR profiling algorithms

21   As shown in Fig.3, D_m can be obtained from DFR* if N_w is known. Likewise, rain rate can

22   be derived if DFR* is used along with Z_{Ku}, as shown in Fig.4. These procedures for the retrieval

23   of D_m and rain rate constitute the basis for the DFR*-based profiling algorithms. On the other

1   hand, the Ku- and Ka-band radar signals undergo losses caused by attenuation of the signals out

2   to the gate at which the radar measurements are made. Accurate attenuation correction is

3   therefore important for the algorithm's overall performance. For down-looking air/space-borne

4   radar, forward and backward recursive procedures are often used for attenuation correction. In

5   the forward approach, the retrieval starts from storm top and then proceeds into storm gate by

6   gate until reaching the surface or final clutter-free gate. Conversely, the backward approach

7   begins at the surface with a known PIA that is usually estimated by the surface reference

8   technique (SRT) or other independent measurements (such as radiometers), with the correction

9   moving upward until reaching the storm top. Benefits and demerits of the forward and backward

10   approaches depend on the types of error sources involved in the algorithms. Nevertheless, it is

11   clear that uncertainties associated with PIA only affect the performance of the backward

12   recursive method while errors associated with attenuation corrections prior to rain, such as

13   attenuation resulting from cloud water and the melting region, are directly related to the

14   performance of the forward recursive method.

15   To retrieve rain and DSD profiles using DFR*, one simple way is to assign the same N_w to

16   each range gate of one vertical profile as an initialization process. With a known N_w, D_m can be

17   uniquely derived from the DFR* if a fixed μ is assumed; rain rate and attenuation coefficients

18   can then be derived by using DFR* along with Z_{Ku}. Attenuation corrections to the measured

19   radar reflectivity profiles are carried out consecutively gate-by-gate along the radar beam either

20   in the forward or backward direction. Since the derived hydrometeor profiles depend on the

21   value of N_w assumed, finding the proper N_w that leads to accurate DSD and rain profiles is a key

22   to the retrieval procedure.

1   In the search for N_w, an optimization method is employed to minimize the discrepancies

2   between the simulated and measured radar parameters. Following Seto et al. (2016) N_w is

3   selected to maximize the product of the following three probability functions (or constraints)

4   from the computations conducted by using a series of initial values of N_w, denoted by $\tilde{N}_{w,k}$,

5   where k=1, 2, …, K (where K is the total number of the initial tests of N_w),

$$p_1(N_w)p_2(N_w)p_3(N_w) = \max(p_1(\tilde{N}_{w,k})p_2(\tilde{N}_{w,k})p_3(\tilde{N}_{w,k})), \quad (12)$$

7   where

$$p_1(\tilde{N}_{w,k}) = \frac{1}{\sigma_1\sqrt{2\pi}}\exp\left(-\frac{(\log_{10}\tilde{N}_{w,k}-3.45)^2}{2\sigma_1^2}\right), \quad (13)$$

$$p_2(\tilde{N}_{w,k}) = \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{(\delta PIA(\tilde{N}_{w,k})-\delta PIA_{SRT})^2}{2\sigma_2^2}\right), \quad (14)$$

$$p_3(\tilde{N}_{w,k}) = \prod_{n=1}^{N}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{(Z_{m,n,est}^{(Ka)}(\tilde{N}_{w,k})-Z_{m,n,obs}^{(Ka)})^2}{2N\sigma_3^2}\right), \quad (15)$$

11   where N is the number of range gates along the column, and δPIA is the differential PIA over the

12   full column defined as difference of PIA between Ka- and Ku-band. $Z_{m,n,est}^{(Ka)}(\tilde{N}_{w,k})$ and $Z_{m,n,obs}^{(Ka)}$

13   are respectively the simulated and measured Ka-band apparent radar reflectivities at the nth

14   range gate. The function $p_1(\tilde{N}_{w,k})$ describes the distribution of $\log_{10}(N_w)$ that has a mean of 3.45

15   in $\log_{10}(mm^{-1}m^{-3})$ and a standard deviation σ_1, derived from the DSD data. $p_2(\tilde{N}_{w,k})$ is

16   associated with the difference in δPIA between the DSD-estimated value and the SRT estimate

17   while $p_3(\tilde{N}_{w,k})$ accounts for discrepancies between the simulated and measured Ka-band

18   reflectivities. Note that $(\sigma_2)^2$ is the error variance associated with the SRT estimate of the

19   differential path attenuation.   The term σ_3 is a constant that determines weighting of

20   measurements of the Ka-band reflectivities in the overall constraint. The smaller this value is, the

21   more the Ka-band measurements are weighted. It is worth mentioning that the reason that δPIA

1   is used rather than the PIA at either or both wavelengths is that the former usually has a smaller

2   error variance than the latter if they are estimated by the SRT because of strong correlation of the

3   normalized surface cross section (σ^0) between Ku- and Ka-band (Meneghini et al. 2015). It is

4   also worth noting that optimization is not required in the standard method since in this case, the

5   DFR is used to estimate D_m directly; N_w is then found from D_m and Z_{Ku} or Z_{Ka}.

6   More detail on the optimization described above is given by the flowchart in Fig.7 which

7   shows the gate-by-gate progression of the retrieval from the rain top to the surface (forward

8   approach). The procedure starts with equally-spaced values of $\log_{10}(\tilde{N}_{w,k})$ (k=1, 2, …, K, where

9   K=100) in the range from 0 to 6, from which the radar and hydrometeor profiles are estimated.

10   The procedure is repeated K times, and the final N_w is chosen as the value that maximizes (12).

11   Once N_w is selected, profiles of D_m and rain rate are found from look-up tables (LUT) that

12   express D_m as a function of N_w and DFR*(γ). Accumulated Ku- and Ka-band attenuations (A_{Ku}

13   and A_{Ka} in dB) are computed from the specific attenuations (k_{Ku} and k_{Ka} in dB/km) of previous

14   gates. The backward approach is done in the same fashion. Unlike the forward approach where

15   the retrieval starts at the rain top where the attenuation is assumed to be negligible, the backward

16   approach begins at the gate near surface where the SRT-derived PIAs are used to estimate the

17   attenuation-corrected reflectivities from the measured radar reflectivities. The backward

18   retrievals are therefore dependent on the accuracy of the PIA estimate.

19   **5.   Evaluation of DFR* Algorithms**

20   Assessments of the modified DFR performance in estimates of rain and D_m are conducted

21   by comparing the estimated results with the true values that are obtained from direct

22   computations from the simulated DSD profiles. Figures 8 and 9 depict, in terms of 2-dimensional

23   probability density functions, the comparisons of D_m and rain rates derived from the modified-

14

1   DFR with γ=0 and 0.7 (left and middle columns) and the standard DFR (right columns) to the

2   corresponding true values of D_m and R. The forward recursive approach, as described in Fig.7,

3   is applied to the non-uniform DSD profiles that are prescribed earlier for these retrievals. To

4   track the statistical errors along the profiles, the comparisons are made at the rain top and the

5   surface that correspond to the beginning and end of the profiles. An unbiased δPIA model with

6   the standard deviation of 0.8 dB, which is similar to what has been found from the GPM DPR

7   measurements over ocean, is used to characterize the PIA errors. The σ_1, σ_2 and σ_3 in (13)-(15)

8   are 3.45 in $\log_{10}$(mm^{-1}m^{-3}), 1.6 in dB (two times of the standard deviation of δPIA) and 2 in dBZ,

9   respectively. Choice of these values seems to lead reasonable and robust estimates after

10   extensive tests. For the standard DFR method, when negative DFR occurs, the larger of the two

11   D_m solutions is chosen. To be consistent with the GPM DPR, the DSD data are used only if the

12   DSD-computed Ku-band reflectivities are greater than 12 dBZ and the Ka-band reflectivities

13   greater than 17 dBZ (DPR sensitivity thresholds). Comparisons of the results derived from the

14   modified and standard DFR methods clearly indicate that the modified DFR delivers far more

15   accurate estimates than does the standard DFR for both rain rate and D_m. The results show

16   generally good correlations between the modified-DFR estimates and the true values despite a

17   slight degradation in accuracy as the retrieval progresses down to the surface. This degradation

18   is the result of accumulating errors in the attenuation correction for ranges deeper into the storm.

19   The rain estimates are more accurate than the D_m estimates because retrieval of D_m is more

20   sensitive to the DSD model assumed, such as selection of μ for the gamma distribution, than is

21   the rain retrieval. It is noticed that there are only slight differences of R and D_m estimates

22   between the γ values of 0 and 0.7. This is because the non-uniform DSD profiles presented in

23   this study don't actually result in significantly large variations of N_w in the profiles. For the case

1   of relatively highly uncorrelated DSD profile, in which the differences in the estimates between

2    $\gamma=0$ and $\gamma=0.7$ are clearly noticeable, i.e., the estimates at $\gamma=0.7$ appear less variable than those at

3    $\gamma=0$ (not shown).

4   As expected, larger uncertainties in the estimates of rain and D_m using the standard DFR

5   take place when the double values occur, as shown in the top-right panels of Figs. 8 and 9. These

6   errors, however, tend to decrease as D_m or rain rate become large; that is, when the DFR-D_m

7   relationship is single-valued.   The ambiguities associated with the double solutions not only

8   adversely affect the retrievals of rain rate and D_m but also introduce errors into the attenuation

9   corrections. This effect can be seen in the bottom-right panels of Figs.8 and 9 where errors

10   associated with the Ku- and Ka-band attenuation corrections become severe enough at the

11   surface to strongly degrade the retrieval estimates, particularly estimates of D_m. The modified

12   and standard DFR comparisons also have been made at the intermediate gates (not shown).

13   These profiles at intermediate gates behave similarly to those in Figs.8 and 9 and are bounded by

14   the results estimated at the beginning and end of the profiles.

15   Figs.10 and 11 show the same retrieval results as in Figs.8 and 9 but for the backward

16   approach. The general behavior of the results obtained from the backward approach largely

17   remains the same as that obtained from the forward approach. Like the forward approach, the

18   modified DFR is superior to the standard DFR method. Because of a reversal of direction of the

19   solution, the last gate of the backward method terminates at the rain top at which the path

20   attenuations from radar are generally small. Unlike the forward method, the accuracy of the PIAs

21   plays a crucial role for the overall performance of the backward procedure.

22   For the results shown in Figs.8-11 the DSDs along the vertical column are taken to be non-

23   uniform, i.e., partially correlated along the column.   If the DSDs are assumed to be fully

1   correlated instead of partially correlated, the rain rate and D_m estimates from the modified DFR

2   tend to improve (not shown). This improvement is the result of the fact that fully correlated

3   DSDs are consistent with the assumption that N_w is constant over the full profile. Changes in σ_1,

4   σ_2 and σ_3 values modify the details of the retrievals but not the general characteristics of the

5   results. The functions p_1, p_2 and p_3 of (13)-(15), serving as constraints on the solutions, play

6   various roles in constraining the results, and their impact on the solutions change with rain

7   intensity. For example, p_1 constitutes a fairly strong constraint in finding the solutions

8   throughout entire rain rate range. p_2 is a weak constraint for rain less than 1 mm/h, a moderate

9   constraint for rain rates between 1 and 10 mm/h, and a strong constraint for rain rates greater

10   than 10 mm/h. p_3 functions in a manner quite similar to p_2 in the sense that it is a strong

11   constraint at moderate and heavy rains, and is a slightly stronger constraint than p_2 for light rain.

12   Overall, the combination of p_1, p_2 and p_3 provide stronger constraints to the retrieval at

13   moderate-heavy rain rates than for light rain. This is because relative errors of δPIA (in p_2 or

14   (13)) become large for the cases where the PIAs are small, which are often associated with the

15   light rain. The Ka-band radar does not provide much additional information if the rain rate is

16   light. In these cases, the backscattering at both Ku- and Ka-band is primarily Rayleigh and the

17   second frequency adds little additional information.

18   **6.   Summary and remarks**

19   Development of a realistic and physically based framework to evaluate dual-wavelength

20   radar techniques from space is important in assessing the accuracy of current and future retrieval

21   algorithms for the dual-wavelength algorithms.   In this study we employ measured DSD data

22   from various NASA-sponsored field campaigns to construct vertical DSD profiles. With these

23   profiles the measured and attenuation-corrected radar reflectivity factors are simulated along

17

1   with the corresponding rain rate and DSD parameters. The simulated radar reflectivities and

2   hydrometeor parameters, because they are directly computed from the measured DSD, are

3   considered as truth. Evaluation of the retrieval algorithms is conducted by comparing the

4   hydrometeor parameters derived from the algorithms that use the DSD-simulated measured

5   reflectivies as inputs with those directly computed from the DSD profiles. For the radar

6   simulations the effect of radar random error and calibration bias are not taken into account.

7   The standard dual-wavelength technique, which relies on DFR-D_m relations, has long been

8   considered as a possible means to derive rain and DSD parameters. However, for the Ku and Ka

9   combination, negative values of DFR lead to ambiguities in the estimation of D_m that can result

10   in large uncertainties in the rain and DSD estimates. If the forward approach is employed, these

11   uncertainties grow larger as the retrieval proceeds deep into the storm as errors are accumulated

12   along the retrieval direction. In an attempt to circumvent the ambiguities in the D_m estimation

13   that the standard dual-wavelength method faces, we introduce a modified version of the standard

14   technique, i.e., the modified dual-wavelength technique, which partially weights the Ka-band

15   measurements in the radar differential-frequency-ratio equations. The modified DFR or DFR*

16   can be viewed as the generalized version of the DFR, from which the standard DFR and the

17   single Ku-band equations are the extreme cases with weighting factors of 1 and 0, respectively.

18   It has been shown that the modified DFR has a unique relation with D_m if the weighting

19   factor (or γ) is less than 0.8 for a given N_w. Unlike the standard DFR (where $\gamma=1$), the DFR*-D_m

20   relations are dependent on N_w. The degree to which the DFR* depends on N_w depends in turn on

21   the value of γ. Generally speaking, dependence of the DFR*-D_m on N_w tends to become weaker

22   as γ increases. From the perspective of radar retrieval, the strategy is to choose as large a γ as

23   possible as long as a unique solution of D_m is achievable. However, a choice of γ near 1 leads to

18

1   an abrupt transition (non-linear behavior) in the DFR*-D_m relations in the range where D_m is less

2   than 1, causing unstable solutions at small values of D_m. Appropriate choice of γ will also

3   depend on some other factors, such as non-uniformity of hydrometeor profiles and model

4   assumptions.

5   To apply the modified DFR for the retrieval of rain profiles, following the work of Seto et

6   al. (2016), an optimal profiling retrieval algorithm is first described and then evaluated using the

7   simulated DSD profiles. This is a recursive procedure that infers hydrometeor parameters and

8   makes attenuation correction gate-by-gate along either forward (storm top, towards the surface)

9   or backward (surface, towards the storm top) along the radar beam. As our primary focus is on

10   examining the mechanics of rain retrieval techniques and assessing their retrieval errors, the

11   uncertainties associated with cloud water and melting hydrometeors that often appear atop the

12   rain as well as the issues related to non-uniform beam filling (NUBF) and multiple scattering are

13   excluded in our evaluation. As such, the attenuation at the first gate for the forward approach

14   (rain top), the gate where the retrieval starts, is assumed to be zero. The PIA is used as the

15   attenuation of the first gate for the backward approach, which is the range gate closest to the

16   surface. The DFR*-based retrievals are carried out K times for a given DSD profile in

17   correspondence with the K equally-spaced initialized $\log_{10}(N_w)$. The final N_w is then determined

18   as that value that maximizes the product of three probability functions that minimize the

19   differences between the radar measurements and the estimates. Once N_w is fixed, the range-

20   profiled rain rate and D_m are derived.

21   It should be pointed out that constant N_w along the vertical profile is an approximation,

22   which is true only if the vertical DSD is uniform. For non-uniform DSD profiles, such as the

23   cases studied in the paper, N_w varies from gate to gate. The constant N_w approximation leads to

1   errors but these errors are limited by the fact that the natural vertical variations in the DSD are

2   not too strong and the fact that the retrieved N_w will be close to a weighted mean of the actual N_w

3   values along the path.

4   Comparisons of the estimates of the rain rate and D_m obtained from the modified and

5   standard DFR to the true values indicate that the DFR*-based profiling retrievals are superior to

6   those from the standard DFR. This is true for both forward and backward approaches. As

7   mentioned earlier, attenuations caused by melting layer and cloud water are not included in our

8   evaluation framework, and errors associated with these attenuations are not taken into account.

9   These errors, if considered, will lead to performance degradation for the forward approach but

10   not for the backward method. The backward approach, however, depends on the accuracy of

11   PIA. Highly unreliable PIA will degrade its performance. It is necessary to point out that for the

12   snow retrieval in which the DFR usually has a unique relation to D_m from the Ku- and Ka-band

13   radar, the standard technique is often advantageous over other approaches because it fully

14   accounts for the particle size distribution (PSD) variations in space and time. This is probably

15   true for other frequency pairs that exhibit either a unique DFR-D_m relation or much-reduced

16   region of DFR where the double solutions occur.

17   The benefits of introducing the DFR* are twofold: varying γ from 0 to 1 coincides with a

18   transition from the single frequency case to the full dual-frequency case; the double-valued

19   solutions obtained when $\gamma=1$ can be mitigated by decreasing γ. Although the DFR* with $\gamma<1$

20   introduces dependency on N_w, it facilitates the use of optimal estimation methods similar to that

21   used in the DPR operational retrieval method. Seeking the most reliable and accurate

22   optimization method, though not the main purpose of this study, is important and will therefore

23   be pursued in future studies.

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1　　　**Figure Captions:**

2

3　　Fig.1 Computational results of D_m vs. DFR (left) and z_{Ku}-normalized rain rate vs. DFR (right) as
4　　the gamma DSD model is assumed with µ values of 0, 3, 6 and 10.

5

6　　Fig.2 Two-dimensional probability density functions, as derived from the DSD measurements
7　　taken during several NASA field campaigns, between D_m and DFR (left) and z_{Ku}-normalized rain
8　　rate vs. DFR (right). Theoretical computations from the gamma DSD model of µ=3 are also
9　　plotted as references.

10

11　　Fig.3　Scatterplots of D_m versus modified DFR as derived from the DSD data for the cases of the
12　　weighting factor γ of the modified DFR equal to 0, 0.5, 0.7 and 1. Colors of the data points
13　　depict N_w values that are derived from the same DSD data. The mean values of the data are given
14　　by the thick dark curves. Theoretical model computations, represented by blue and red curves,
15　　are provided as the DSD is modeled as the gamma distributions with a fixed µ of 3 and N_w (blue
16　　curves) ranging from 10 to 10^5 in $mm^{-1}m^{-3}$. Contours of Z_{Ku} at the values from 0 to 40 dB (red
17　　curves) are plotted as well for reference.

18

19　　Fig.4　Scatterplots of Z_{Ku}-normalized rain rate versus modified DFR as derived from the DSD
20　　data for the cases of the weighting factor γ of the modified DFR equal to 0, 0.5, 0.7 and 1. Colors
21　　of the data points depict N_w values derived from the same DSD data. The mean values of the data
22　　are given by the thick dark curves. Theoretical model computations, represented by blue and red
23　　curves, are provided as the DSD is modeled as the gamma distributions with a fixed µ of 3 and
24　　N_w ranging from 10 to 10^5 in $mm^{-1}m^{-3}$. Contours of Z_{Ku} at the values from 0 to 40 dB (red
25　　curves) are plotted as well for reference.

26

27　　Fig.5　Ku-band radar reflectivities (top panels) and rain rate (bottom panels) simulated from a
28　　segment of the DSD profiles generated from measured DSD data. Homogeneously vertical DSD
29　　profiles are given in the left column while the non-uniform DSD profiles are provided in the
30　　right column.

31

32　　Fig.6　Averaged spatial correlations of rain rate, D_m and $\log_{10}N_w$ for the non-uniform DSD
33　　profiles between radar range heights and surface. Results from three rain ranges, i.e., R<5, R≥5
34　　and R>0 mm/h (all rain), are provided for approximately representing stratiform, convective and
35　　their combined systems.

36

37　　Fig.7　Flowchart of the modified dual-wavelength technique (DFR*) for retrieval of DSD
38　　parameters and rain rate. Outer-loop variable k goes from 1 to K to provide equal-spaced values
39　　of $\log_{10}(\tilde{N}_{w,k})$ at each step while inner-loop variable n, running from 1 to N, represents radar
40　　range gates at which the retrievals are carried out.

41

42　　Fig.8　Two-dimensional probability density functions of D_m estimated by the modified DFR
43　　with γ of 0 and 0.7 (left and middle columns) as well as the standard DFR (γ=1) (right column)
44　　with the true D_m for the range gate at the rain top (top row) and at the surface (bottom row).　The
45　　one-to-one lines are given by the black solid curves while the means and twice the standard

1 deviations of the data are given by the white solid curves and vertical bars, respectively. For
2 both the modified and standard DFR methods, the forward recursive approach is used and
3 applied to the non-uniform DSD profiles. An unbiased statistical δPIA model with the standard
4 deviation of 0.8 dB is assumed.
5
6 Fig.9   Two-dimensional probability density functions of rain rate estimated by the modified
7 DFR with γ of 0 and 0.7 (left and middle columns) as well as the standard DFR (γ=1) (right
8 column) with the true rain rate for a range gate at the rain top (top row) and at the surface
9 (bottom row). The one-to-one lines are given by the black solid curves while the means and
10 twice the standard deviations of the data are given by the white solid curves and vertical bars,
11 respectively. The non-uniform DSD profiles are assumed and the forward recursive approach is
12 applied for both the standard and modified DFR methods.   An unbiased statistical δPIA model
13 with the standard deviation of 0.8 dB is assumed.
14
15 Fig.10   Two-dimensional probability density functions of D_m estimated by the modified DFR
16 with γ of 0 and 0.7 (left and middle columns) as well as the standard DFR (γ=1) (right column)
17 with the true D_m for a range gate at the rain top (top row) and at the surface (bottom row). The
18 one-to-one lines are given by the black solid curves while the means and twice the standard
19 deviations of the data are given by the white solid curves and vertical bars, respectively. The
20 backward recursive approach is applied in all cases to the non-uniform DSD profiles. Unbiased
21 statistical PIA and δPIA models with the standard deviations of 2 dB and 0.8 dB are assumed,
22 respectively.
23
24 Fig.11   Two-dimensional probability density functions of rain rate estimated by the modified
25 DFR with γ of 0 and 0.7 (left and middle columns) as well as the standard DFR (γ=1) (right
26 column) with the true rain rate for a range gate at the rain top (top row) and at the surface
27 (bottom row). The one-to-one lines are given by the black solid curves while the means and
28 twice the standard deviations of the data are given by the white solid curves and vertical bars,
29 respectively. The backward recursive approach is applied to the non-uniform DSD profiles.
30 Unbiased statistical PIA and δPIA statistical models with the standard deviations of 2 dB and 0.8
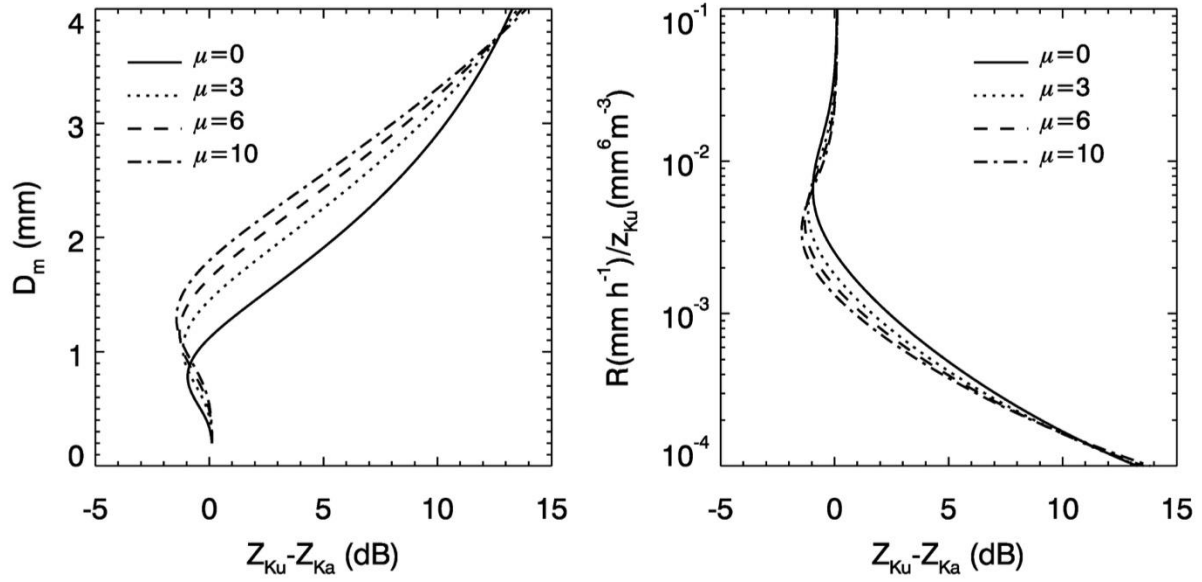31 dB are assumed, respectively.
32

Fig.1 Computational results of D_m vs. DFR (left) and z_{Ku}-normalized rain rate vs. DFR (right) as the gamma DSD model is assumed with μ values of 0, 3, 6 and 10.
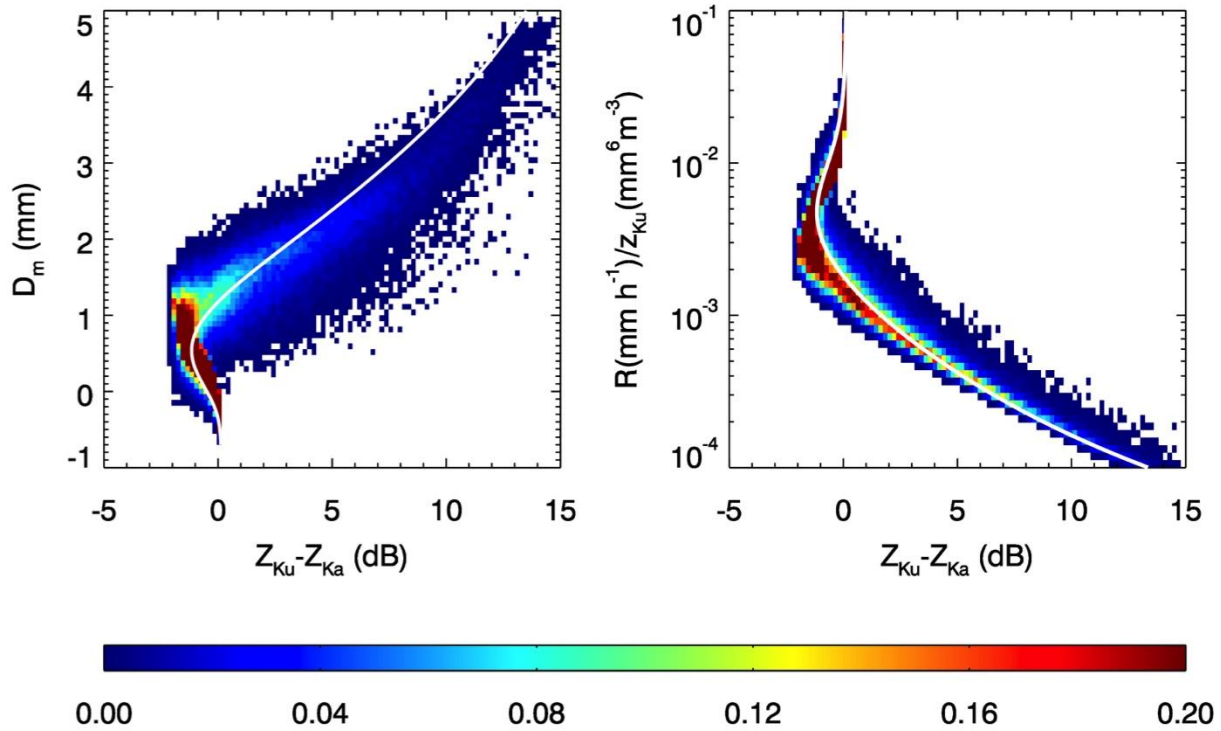
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5 Fig.2 Two-dimensional probability density functions, as derived from the DSD measurements
6 taken during several NASA field campaigns, between D_m and DFR (left) and z_{Ku}-normalized rain
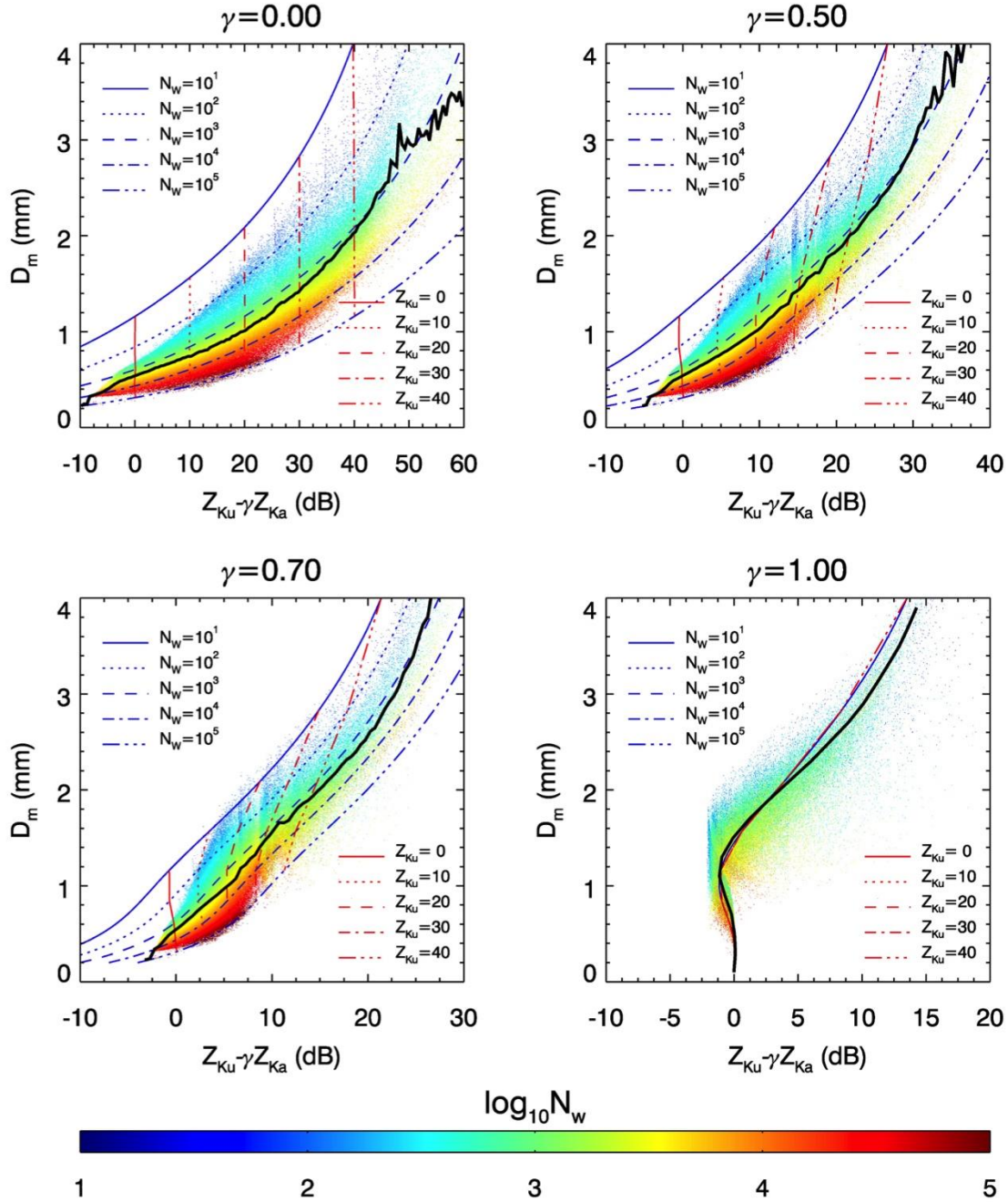7 rate vs. DFR (right). Theoretical computations from the gamma DSD model of $\mu=3$ are also
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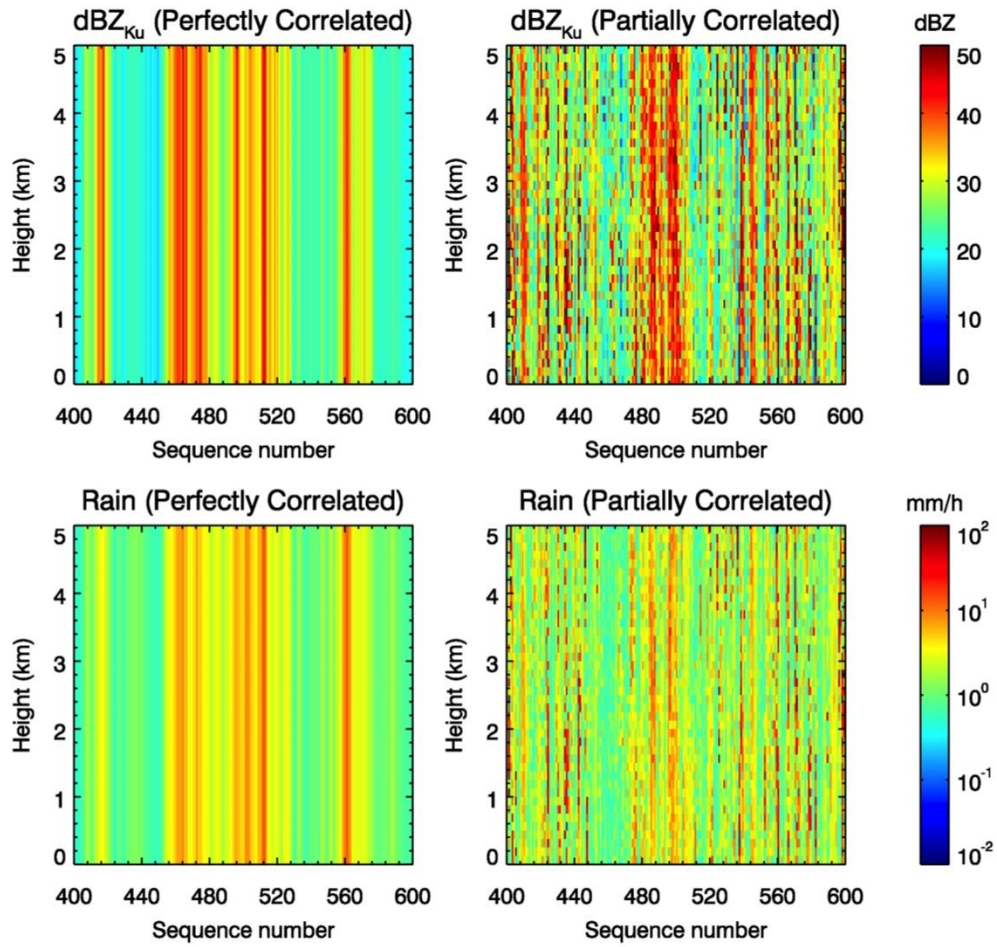curves) are plotted as well for reference.

Fig.4 Scatterplots of Z_{Ku}-normalized rain rate versus modified DFR as derived from the DSD data for the cases of the weighting factor γ of the modified DFR equal to 0, 0.5, 0.7 and 1. Colors of the data points depict N_w values derived from the same DSD data. The mean values of the data are given by the thick dark curves. Theoretical model computations, represented by blue and red curves, are provided as the DSD is modeled as the gamma distributions with a fixed μ of 3 and N_w ranging from 10 to 10^5 in $mm^{-1}m^{-3}$. Contours of Z_{Ku} at the values from 0 to 40 dB (red curves) are plotted as well for reference.

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4 Fig.5 Ku-band radar reflectivities (top panels) and rain rate (bottom panels) simulated from a
5 segment of the DSD profiles generated from measured DSD data. Homogeneously vertical DSD
6 profiles are given in the left column while the non-uniform DSD profiles are provided in the
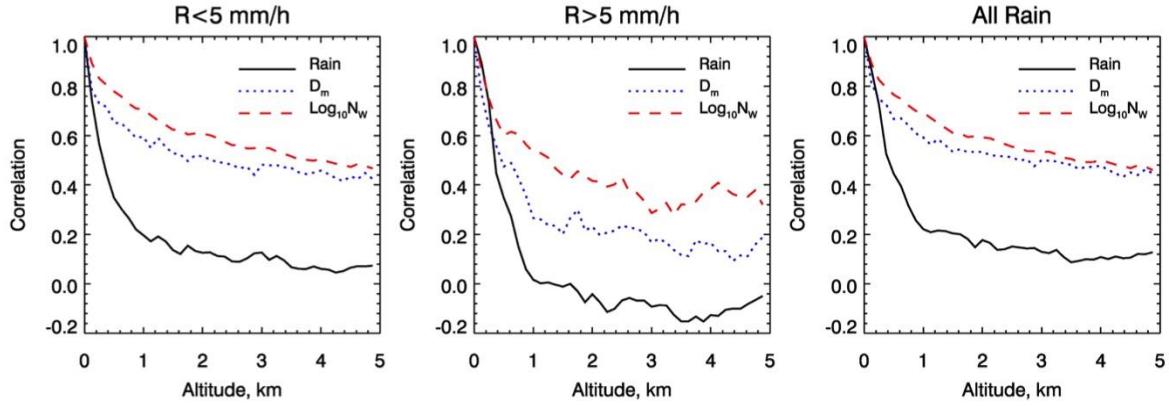7 right column.
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R<5 mm/h   R>5 mm/h   All Rain

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8 Fig.6 Averaged spatial correlations of rain rate, D_m and $\log_{10}N_w$ for the non-uniform DSD
9 profiles between radar range heights and surface. Results from three rain ranges, i.e., R<5, R≥5
10 and R>0 mm/h (all rain), are provided for approximately representing stratiform, convective and
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12

Fig.7 Flowchart of the modified dual-wavelength technique (DFR*) for retrieval of DSD
parameters and rain rate. Outer-loop variable k goes from 1 to K to provide equal-spaced values
of $\log_{10}(\tilde{N}_{w,k})$ at each step while inner-loop variable n, running from 1 to N, represents radar
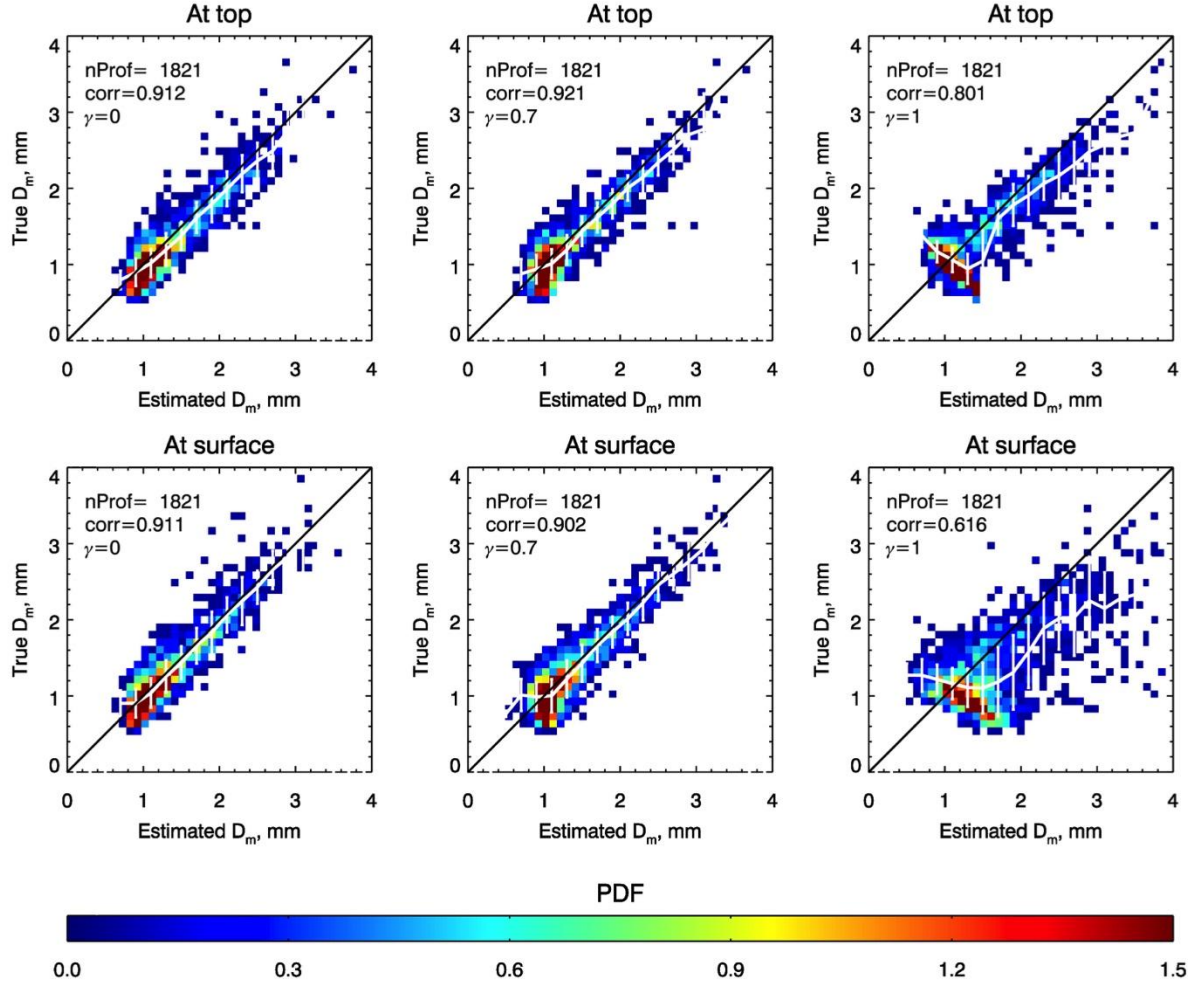range gates at which the retrievals are carried out.

Fig.8   Two-dimensional probability density functions of D_m estimated by the modified DFR with γ of 0 and 0.7 (left and middle columns) as well as the standard DFR (γ=1) (right column) with the true D_m for the range gate at the rain top (top row) and at the surface (bottom row). The one-to-one lines are given by the black solid curves while the means and twice the standard deviations of the data are given by the white solid curves and vertical bars, respectively. For both the modified and standard DFR methods, the forward recursive approach is used and applied to the non-uniform DSD profiles. An unbiased statistical δPIA model with the standard deviation of 0.8 dB is assumed.
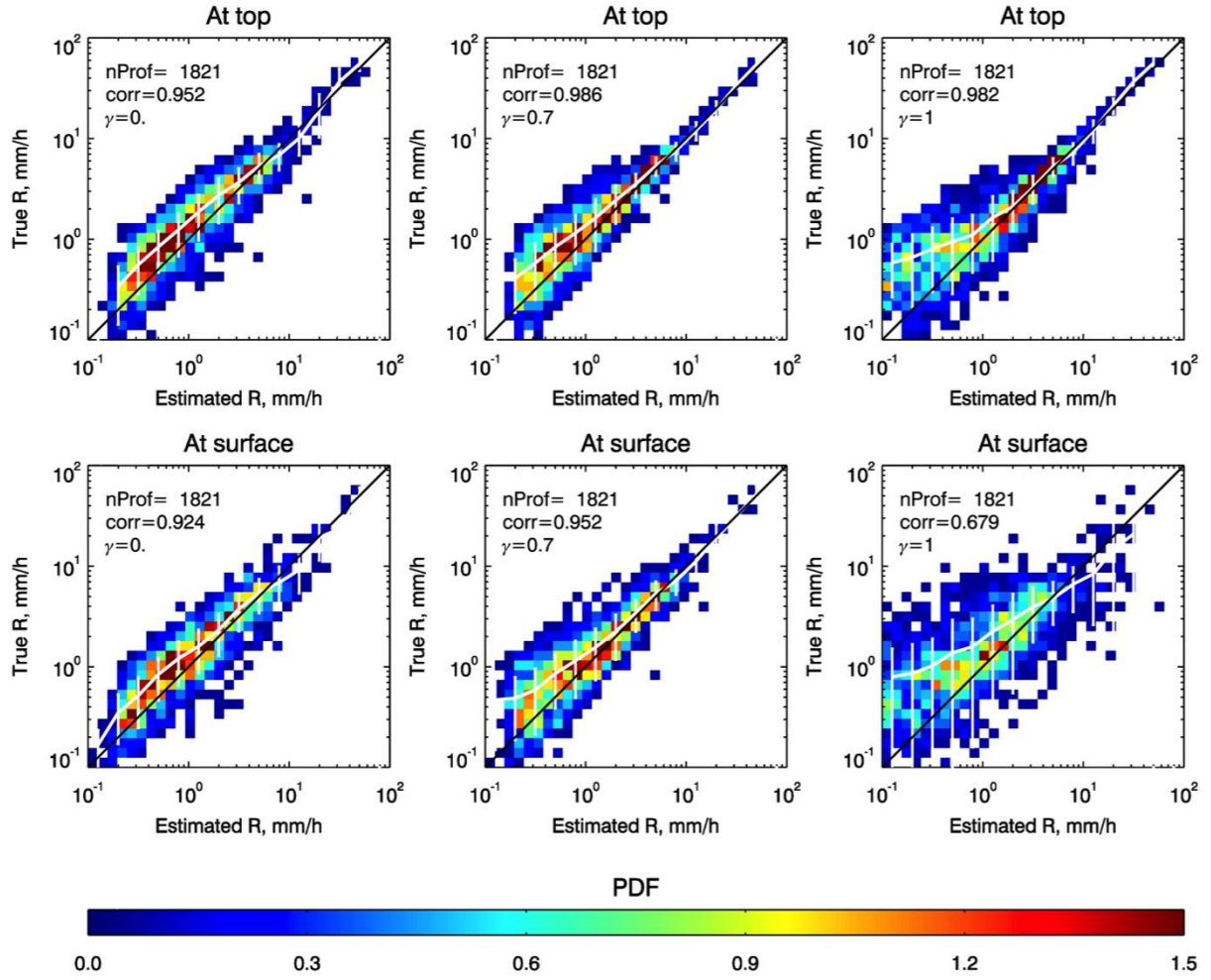
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Fig.9   Two-dimensional probability density functions of rain rate estimated by the modified
DFR with γ of 0 and 0.7 (left and middle columns) as well as the standard DFR (γ=1) (right
column) with the true rain rate for a range gate at the rain top (top row) and at the surface
(bottom row). The one-to-one lines are given by the black solid curves while the means and
twice the standard deviations of the data are given by the white solid curves and vertical bars,
respectively. The non-uniform DSD profiles are assumed and the forward recursive approach is
applied for both the standard and modified DFR methods.   An unbiased statistical δPIA model
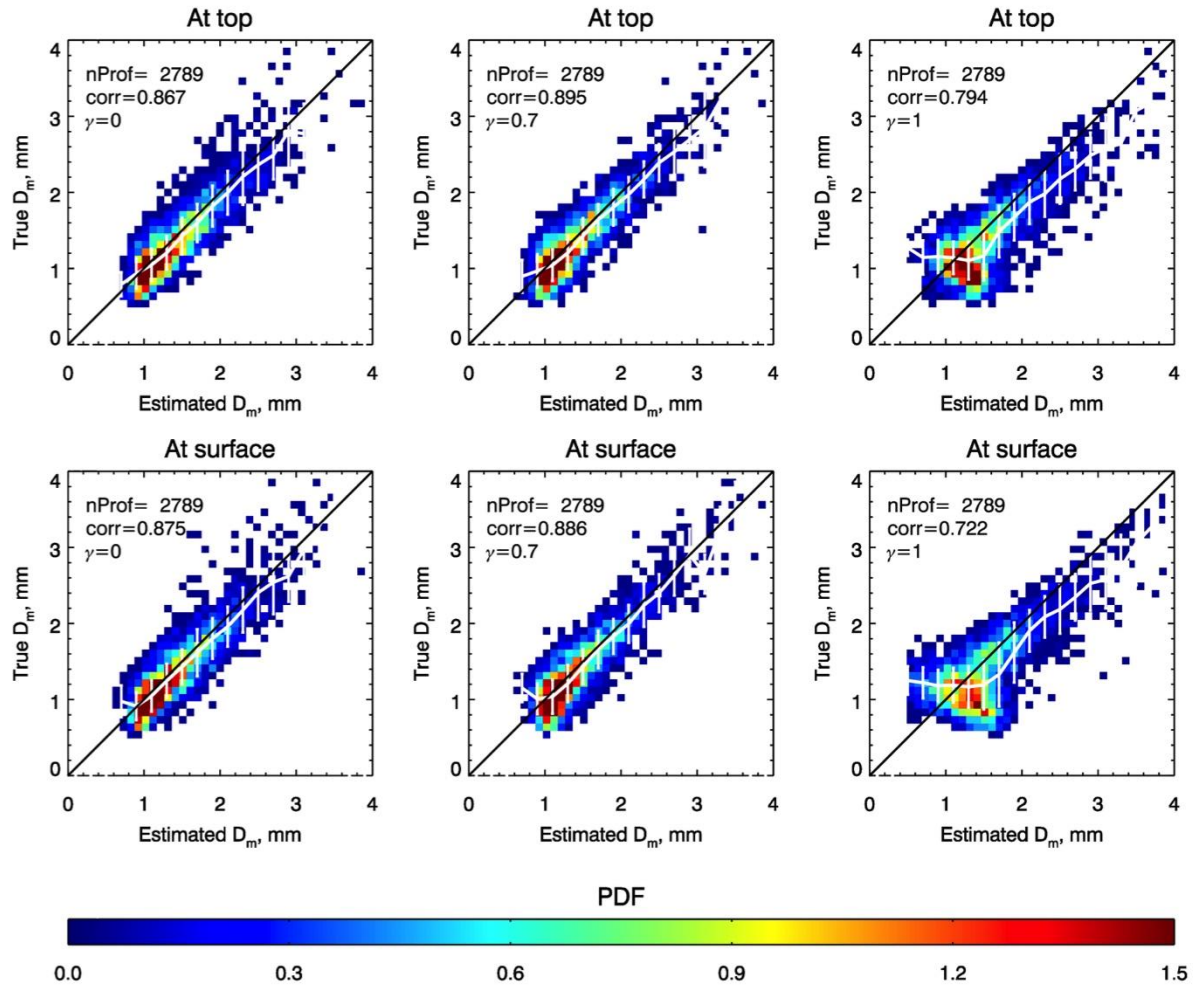with the standard deviation of 0.8 dB is assumed.

Fig.10   Two-dimensional probability density functions of D_m estimated by the modified DFR with γ of 0 and 0.7 (left and middle columns) as well as the standard DFR ($\gamma=1$) (right column) with the true D_m for a range gate at the rain top (top row) and at the surface (bottom row). The one-to-one lines are given by the black solid curves while the means and twice the standard deviations of the data are given by the white solid curves and vertical bars, respectively. The backward recursive approach is applied in all cases to the non-uniform DSD profiles. Unbiased statistical PIA and δPIA models with the standard deviations of 2 dB and 0.8 dB are assumed, respectively.
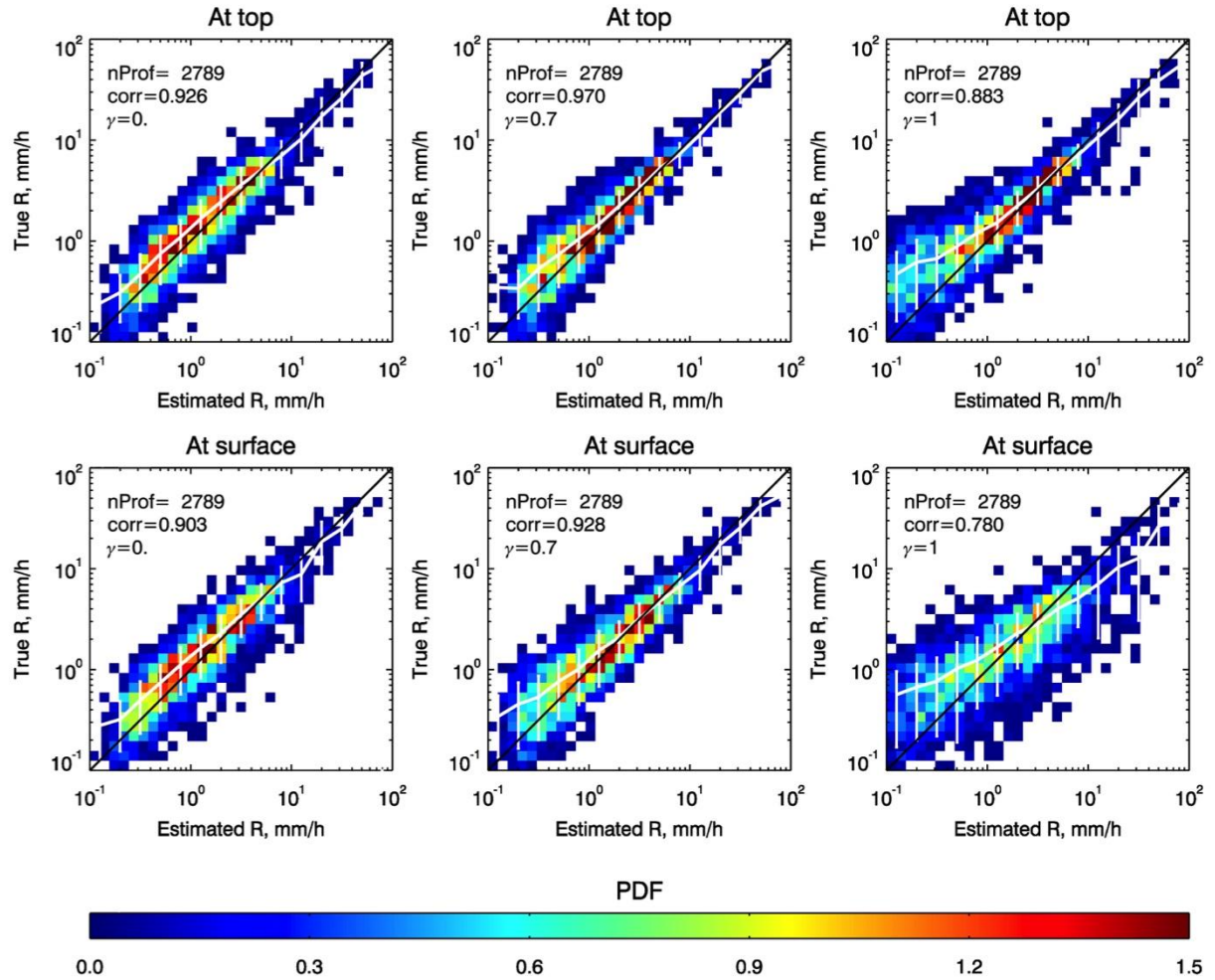
Fig.11 Two-dimensional probability density functions of rain rate estimated by the modified DFR with γ of 0 and 0.7 (left and middle columns) as well as the standard DFR ($\gamma=1$) (right column) with the true rain rate for a range gate at the rain top (top row) and at the surface (bottom row). The one-to-one lines are given by the black solid curves while the means and twice the standard deviations of the data are given by the white solid curves and vertical bars, respectively. The backward recursive approach is applied to the non-uniform DSD profiles. Unbiased statistical PIA and δPIA statistical models with the standard deviations of 2 dB and 0.8 dB are assumed, respectively.